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FOREIGN TECHNOLOGY DIVISION

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by

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THE STUDY OF SUPERSONIC FLOW AROUND DELTA WINGS WITH FORCED ANTISYMMETRY TAKING INTO CONSIDERATION THE FALLING OFF OF FLOW AT THE LEADING EDGES

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This paper studies the supersonic flow around thin delta wings wings having forced antisymmetric distribution of incidences, taking into consideration the separation of flow at the leading edges. Considering an imaginary thin wing, equivalent to a real one from an aerodynamic point of view, the distribution of pressure and aerodynamic characteristics are determined.

1. PRELIMINARY CONSIDERATIONS

In that which follows we will do a study of the supersonic flow around thin delta wings having forced antisymmetric distribution of incidences, taking into consideration the falling off of flow at its subsonic leading edges. The antisymmetric distribution of the incidences or of the vertical velocities corresponds to a forced antisymmetric curved delta wing according

to a liniar function or an antisymmetric deflection.

While the incidences are very small, the pressure on the wing follows approximately the stability laws in the hypothesis of small disturbances, outside the system in immediate proximity to the leading edges, where the corresponding velocity and pressure are finite, though infinite values would result from liniar theory. We can say therefore, that outside this area limited by the angle of the leading edge, this theory is valid for the whole wing, such that the falling off of flow becomes hardly felt and this influence is small on the whole contents of the wing.

As the values of the size of the local incidences become greater, the flow separates at the leading edge, as with the plane delta wing with constant incidence, creating a vortex layer which sits above and below the wing, producing an antisymmetric movement. The vortex layer, having sufficiently small thickness, can be considered a vortex sheet which is rolled up in the form of a horn, composed of a concentrated nucleus and a marginal vortex sheet, starting at the leading edge.

The incidence being variable on the wing surface, the axis on which the horn is wrapped will be a curve, and the vortex generation intensity of the nucleus is variable along the axis proportional with the square of the opening of the wing. For

simplification, in that which follows, the axis on which the nucleus of the vortex is situated is considered to be a straight line. Thus the system of two concentrated vortexes, of the same intensity and sign, situated antisymmetrically in reference to the axis of symmetry O_{x_1} (fig. 1) at the abscissa c and the ordinate t, will bring essential modifications on the field of flow around the wing.

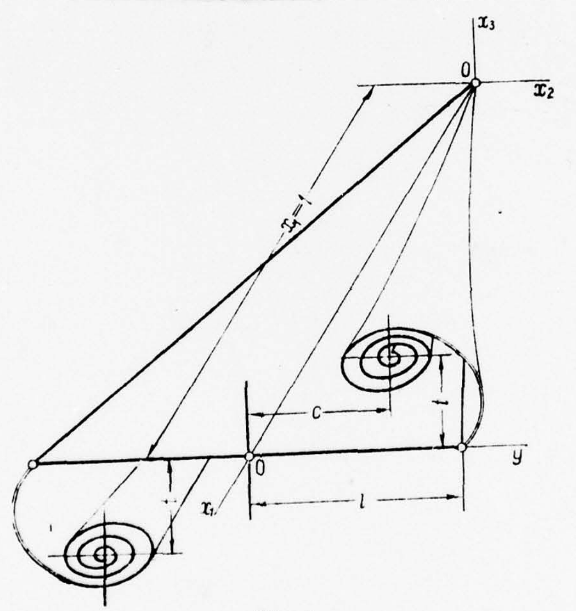

Fig. 1

The resulting flow, which will become more complicated, will be studied on the basis of the conical theory of motion of the second order (1).

For this we will follow the direction used in prior works (2) - (4), where solutions were given the the thin delta wing

with constant and antisymmetric incidence, respectively, (forced antisymmetry) in reference to the axis Ox_1, which led toward conical motion by the first order (conical as a matter of fact).

We will allow that the effect of the falling off of flow at the edge of the wing and the formation of the two antisymmetric nucleuses consists of the modification of the field of vertical and longitudinal velocities, having as a result the avoidance of infinite velocities at the leading edge, as results from the classical liniar theory. But it can be allowed that the effect of the longitudinal velocities of disturbance can be substituted through a corresponding distribution of the vertical velocities. By that we will consider a distribution by incidences or by vertical velocities, forced antisymmetrically, so as to correspond to a real case of an imaginary thin delta wing with variable incidence, different on both its sides, having at the same time finite velocities at the leading edge.

It will be allowed that the real thin wing, which has in a certain way finite velocities at the edge through the effect of the separation of flow, is equivalent from an aerodynamic point of view with an imaginary thin wing, having the same variation of incidences which I defined earlier.

In order to study the flow more easily through the conical

motion method, we will take apart the imaginary wing corresponding to distribution of the above vertical velocities into three wing components, in the same way as in (2) - (4):

1) the thin wing, having a variation of forced antisymmetric incidence suitably chosen in order to follow in some measure the phenomena of the modification of pressures and of the vertical velocities of disturbance on the surface of the wing near the leading edge. Thus an imaginary thin wing is obtained with finite velocity at the leading edge, but equal and of opposed direction on the two sides, higher and lower;

2) The wing of "symmetrical" thickness, having variable slope in the same way as the incidence of the first wing. This wing, combined with the first, will form a wing with different pressures on the two sides, as it is in reality;

3) The third wing will have symmetrical "thickness", with variable slope and forced antisymmetry, however in such a way that, combined with the wing from 2), a nought mean thickness is obtained, characteristic of a real thin wing. Superpositioning the flow around the three wing components, we will obtain the resulting imaginary wing, equivalent from an aerodynamic point of view with the delta wing with the separation of the flow at the edge.

2. THE DETERMINATION OF THE AXIS OF DISTURBANCE VELOCITIES

In continuation we will follow the way of determining the axis of disturbance velocities for the three wing components with forced antisymmetry, being necessary for the determination of the distribution of pressures and of the aerodynamic characteristics of the resulting imaginary wing, which are presupposed to be the same as real thin delta wings, having the incidence defined by the relation

$$w = \pm w_{10}\, x_1 = \mp x_1\, \alpha_{10} U_\infty. \tag{1}$$

We will note further

$$w'_u = -\alpha'_u U_\infty, \quad w'_l = -\alpha'_l U_\infty, \tag{2}$$

the vertical velocities and the incidences w'_u, α'_u respectively on the higher surface, w'_l, α'_l on the lower of the thin imaginary wing.

The movement around the wing being conical by the second order, we will use the same method used, considering in this direction the physical plane Oyz (fig. 1) normal on the axis Ox_1 and having the coordinates

$$y = \frac{x_2}{x_1}, \quad z = \frac{x_3}{x_1}, \tag{3}$$

the axis Oy and Oz being parallel, with Ox_2 and Ox_3 respectively. Further we will make a similar transformation with that given by Busemann (fig. 2):

$$\mathfrak{y} = \frac{y}{1 - B^2 z^2}, \quad \mathfrak{z} = \frac{z\sqrt{1 - B^2(y^2 + z^2)}}{1 - B^2 z^2} \quad (x = \mathfrak{y} + i\mathfrak{z}), \qquad (4)$$

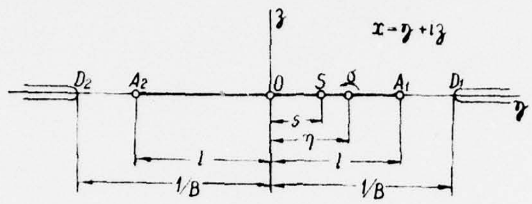

Fig. 2

obtaining a plane which has the property of keeping the track of the wing ($y = \mathfrak{y}$, $z = \mathfrak{z} = 0$) in the true magnitude. In this plane, the first derivative of the disturbance velocities u, v, and w are harmonic functions and can be associated to corresponding conjugated functions, in such a way as to obtain variable complex analytic functions:

$$x = \mathfrak{y} + i\mathfrak{z}. \qquad (5)$$

We will study each wing defined above in turn.

2.1. **The antisymmetric thin wing.** As a result of the effects of the two nucleuses of the vortexes, the vertical velocity on the real wing is modified, as well as the first wing component, defined above.

Thus, for the points contained between ($-s<y<s$) for the track of the wing contained in the plane $x = \mathfrak{y} + i\mathfrak{z}$ (5), the vertical velocity will be considered constant for x_1 = constant

$$w' = \pm x_1 w_{10}'^{(0)} = \mp x_1 \alpha_{10}^{(0)} U_\infty, \qquad (6)$$

where the parameter $w_{10}^{(0)}$ corresponds to the abscissa $y = s$, and for the area $y \in [-1,-s] \cup [s,1]$ we will write

$$w' = \pm x_1 w_{10}'(y) = \mp x_1 \alpha_{10}'(y) U_\infty, \qquad (7)$$

such that at the leading edge ($y = \pm 1$) will be obtained

$$w^{(1)} = \pm x_1 w_{10}^{(1)} = \mp x_1 \alpha_{10}^{(1)} U_\infty . \qquad (8)$$

This continuing variation of vertical velocities (or, more precisely, of the parameter $w_{10}'(y)$) corresponds to the continuing distributions of elementary edges, situated on the wing in the interior of the considered space, which gives each point $y = \eta$ the elementary drop.

However, taking into consideration previous works [1], [2], the contribution of elementary edges situated in the point $y = \eta$ in the expression of the axis of disturbance velocity in the point x, applying the similar hydrodynamic method, it will be

$$d\mathcal{U} = q_{20}' \cosh^{-1} \sqrt{\frac{(l+\eta)(l-x)}{2l(\eta - x)}}\, d\eta. \qquad (9)$$

These contributions of the edges in the expressions of the axis of disturbance and vertical velocities on the wing are realized placing on the trace of the wing from the plane $x = y + i\gamma$ (5) some singularities (sources) by order two.

In this way we can get in calculation the effect of the two nucleuses of the vortexes concentrated among the continuing distribution of sources. This division will be necessary to satisfy the conditions imposed by the problem of obtaining concomitant axis of disturbance and vertical velocities indicated above on the basis of observations and the resulting experiments. From that we will chose a liniar variation of intensities of sources:

$$q'(y) = q^* \left(1 - \frac{|\eta|}{l}\right) \quad (s < \eta < l), \tag{10}$$

which, in the case of homogenous motion of the second order (n = 2), is written

$$q'_{20} = \pm\, q^*_{20}\left(1 - \frac{|\eta|}{l}\right), \quad q'_{21} = \pm\, q^*_{21}\left(1 - \frac{|\eta|}{l}\right). \tag{11}$$

The axis of disturbance velocity for the thin wing component will be obtained through the addition of the contributions of all the elementary distribution edges and of the subsonic edges [1] under the form

$$\begin{aligned} \frac{1}{r_1}\mathcal{U}_1 = \mathcal{U}_{1l} = {} & \frac{A_{21}\, x}{\sqrt{l^2 - x^2}} + \frac{2}{\pi} Q^*_{21}\, x \cosh^{-1}\sqrt{\frac{l^2}{x^2}} + \\ & + \frac{2}{\pi}\int_s^l \left(1 - \frac{\eta}{l}\right)(q^*_{20} + x q^*_{21}) \cosh^{-1}\sqrt{\frac{(l+\eta)(l-x)}{2l(\eta - x)}}\, d\eta - \\ & - \frac{2}{\pi}\int_s^l \left(1 - \frac{\eta}{l}\right)(q^*_{20} - x q^*_{21}) \cosh^{-1}\sqrt{\frac{(l+\eta)(l+x)}{2l(\eta + x)}}\, d\eta, \end{aligned} \tag{12}$$

which, after the accomplishment of the calculations, becomes

$$\begin{aligned} \frac{1}{r_1}\mathcal{U}_1 = {} & \frac{A_{21}\, x}{\sqrt{l^2 - x^2}} + \frac{2}{\pi} Q^*_{21}\, x \cosh^{-1}\sqrt{\frac{l^2}{x^2}} - \\ & - \frac{1}{\pi}\Big[(q^*_{20} + q^*_{21}\, x)(s - x)\left(1 - \frac{s + x}{2l}\right)\cosh^{-1}\frac{l^2 - sx}{l(s - x)} - \end{aligned}$$

$$-(q_{20}^{*} - q_{21}^{*}x)(s+x)\left(1 - \frac{s-x}{2l}\right)\cosh^{-1}\frac{l^2+sx}{l(s+x)} +$$

$$+ x\left[(q_{20}^{*} - 2q_{21}^{*}l)\cos^{-1}\frac{s}{l} + q_{21}^{*}l\sqrt{1-\frac{s^2}{l^2}}\right]\sqrt{1-\frac{x^2}{l^2}}\Bigg], \quad (13)$$

in which $A_{21}, Q_{21}^{*}, q_{10}^{*}, q_{21}^{*}$ are some constants which will be determined below, $\frac{2}{\pi}Q_{21}^{*}$ being the intensity of a source sitting in the origin (x = 0), due to the central edges which appear at the wing with forced antisymmetry.

2.2. The wing of symmetrical thickness having equal slope with the incidence of the first thin wing. Through the introduction of this wing of symmetrical thickness, the accentuated peaks of pressure on the lower side of the wing is removed, where the distribution of pressure obtained through the superpositioning with the first wing component will be different from that on the higher side. Following the general method of conical motion [1], for a wing of symmetrical thickness with the variation of slope given by the same distribution of sources (11) we will write the following expression for the axis of disturbance velocity:

$$\frac{1}{x_1}\mathcal{U}_{t} = \mathcal{U}_{t1} = \frac{2}{\pi}\int_s^l\left(1-\frac{\eta}{l}\right)(q_{20}^{*} + xq_{21}^{*})\cos h^{-1}\sqrt{\frac{(1+B\eta)(1-Bx)}{2B(\eta - x)}}\,d\eta -$$

$$-\frac{2}{\pi}\int_s^l\left(1-\frac{\eta}{l}\right)(q_{20}^{*} - xq_{21}^{*})\cos h^{-1}\sqrt{\frac{(1+B\eta)(1+Bx)}{2B(\eta + x)}}\,d\eta +$$

$$+\frac{2}{\pi}Q_{21}^{**}x\cos h^{-1}\sqrt{\frac{1}{B^2x^2}} \pm L. \quad (14)$$

Accomplishing the integrals above, we will find

$$\frac{1}{x_1}\mathcal{U}_{t} = \frac{1}{\pi}\left\{(q_{20}^{*} + q_{21}^{*}x)\left[(l-x)\left(1-\frac{l+x}{2l}\right)\cos h^{-1}\frac{1-B^2lx}{B(l-x)} -\right.\right.$$

$$- (s - x)\left(1 - \frac{s + x}{2l}\right)\cos h^{-1}\frac{1 - B^2 s x}{B(s - x)}\Bigg] -$$

$$- (q_{20}^{*} - q_{21}^{*} x)\left[(l + x)\left(1 - \frac{l - x}{2l}\right)\cos h^{-1}\frac{1 + B^2 l x}{B(l + x)} -\right.$$

$$- (s + x)\left(1 - \frac{s - x}{2l}\right)\cos h^{-1}\frac{1 + B^2 s x}{B(s + x)}\Bigg] -$$

$$- \frac{x\sqrt{1 - B^2 x^2}}{Bl}\Big[(\sin^{-1} Bl - \sin^{-1} Bs)(q_{20}^{*} - 2q_{21}^{*} l) -$$

$$- \frac{q_{21}^{*}}{B}(\sqrt{1 - B^2 l^2} - \sqrt{1 - B^2 s^2})\Big]\Big\} + \frac{2}{\pi} Q_{21}^{**} x \cos h^{-1}\sqrt{\frac{1}{B^2 x^2}} \pm L, \quad (15)$$

where L represents the contribution of the subsonic edge having the slope equal with $\alpha_{10}^{(1)} x_1$, and the term which presents the coefficient $\frac{2}{\pi} Q_{21}^{**}$ appears only in the case of forced antisymmetry.

2.3. The wing of symmetrical thickness compensates for slope. The introduction of the effect of the wing from point 2.2 makes the resulting wing have "symmetrical thickness". In order to compensate this work, we will introduce a new distribution source of a certain form, which will restore the wing to a mean nought thickness. The variation of the vertical velocities w'' given by these sources will correspond to a "compensating wing slope", having still a symmetrical thickness in reference with the axis Ox_3, and antisymmetric face for Ox_2. This wing, having at the edge of the wing the velocity $-w_{10}^{(1)} x_1$, will cancel the mean slope and the effect L of the slope of the edge of the wing 2.2. The distribution of sources of intensity q'' will be necessary to create on the lower side

of the wing a distribution of pressure without accentuated peaks, approximately liniar, with the exception of the points near the leading edges.

To simplify, we will take the following expression of the distribution of the intensities of the sources:

$$q''_{20} = q^{**}_{20}\frac{\eta}{l}, \quad q''_{21} = q^{**}_{21}\frac{\eta}{l}, \quad (-l \leqslant \eta \leqslant l). \tag{16}$$

We thus obtained two "large wings" in order to form a single one, having the slope variable in such a way for the mean to be nought.

The axis of disturbance velocity U_{1c} for the third wing will be the following:

$$\frac{1}{x_1}\mathcal{U}_c = \mathcal{U}_{1c} = \frac{2}{\pi} q^{**}_{20}\int_0^l \frac{\eta}{l}\left(\cos h^{-1}\sqrt{\frac{(1+B\eta)(1-Bx)}{2B(\eta-x)}} - \right.$$
$$\left. -\cos h^{-1}\sqrt{\frac{(1+B\eta)(1+Bx)}{2B(\eta+x)}}\right)\mathrm{d}\eta + x\frac{2}{\pi}q^{**}_{21}\int_0^l \frac{\eta}{l}\left(\cos h^{-1}\sqrt{\frac{(1+B\eta)(1-Bx)}{2B(\eta-x)}} + \right.$$
$$\left. + \cos h^{-1}\sqrt{\frac{(1+B\eta)(1+Bx)}{2B(\eta+x)}}\right)\mathrm{d}\eta + \frac{2}{\pi}Q^{***}_{21}x\cos h^{-1}\sqrt{\frac{1}{B^2x^2}} \mp L. \tag{17}$$

In the course of the calculations we are led to the expression

$$\frac{1}{x_1}\mathcal{U}_c = \frac{1}{2\pi B^2 l}\left\{B^2(l^2-x^2)\left[(q^{**}_{20} + q^{**}_{21}x)\cosh^{-1}\frac{1-B^2lx}{B(l-x)} - \right.\right.$$
$$\left. -(q^{**}_{20} - q^{**}_{21}x)\cosh^{-1}\frac{1+B^2lx}{B(l+x)}\right] +$$
$$+2Bx\left[\left(q^{**}_{20}\sin^{-1}Bl + \frac{q^{**}_{21}}{B}(1-\sqrt{1-B^2l^2})\right)\sqrt{1-B^2x^2} + \right.$$
$$\left.\left. +q^{**}_{21}Bx^2\cos h^{-1}\sqrt{\frac{1}{B^2x^2}}\right]\right\} + \frac{2}{\pi}Q^{***}_{21}x\cos h^{-1}\sqrt{\frac{1}{B^2x^2}} \mp L, \tag{18}$$

where the term $\frac{2}{\pi} Q_{21}^{***}$ corresponds only to forced antisymmetry.

Superpositioning the three wing components, the resulting imaginary wing is obtained, equivalent from an aerodynamic point of view with the real wing, for which the axis of disturbance velocity has the expression

$$\mathcal{U}_1 = \mathcal{U}_{1l} + \mathcal{U}_{1s} + \mathcal{U}_{1c}, \tag{19}$$

which will be the antisymmetrical face of the axis of symmetry Ox_1 and continues in the origin O. We will observe that the velocity U_{11} on the higher surface is equal and of the opposed sign with that of the lower surface, as corresponds to the thin wing.

3. DETERMINATION OF THE CONSTANTS

For the determination of the constants q_{20}^{*} and q_{21}^{*} which appear in the expression (13) of U_{11}, we will start from the following conditions:

$$\operatorname{Im} \int_{\sigma} \sqrt{1 - B^2 x^2} \frac{d^2}{dx^2} \left(\frac{d\mathcal{U}_{11}}{d\eta} \right) dx = \frac{dw'_{10}}{d\eta}, \tag{20}$$

$$-\operatorname{Im} \int_{\sigma} \frac{\sqrt{1 - B^2 x^2}}{x} \frac{d^2}{dx^2} \left(\frac{d\mathcal{U}_{11}}{d\eta} \right) dx = 0, \tag{20'}$$

deduced from the theory of conical motion [1], the integration being made on a semicircle σ of very small radius around a certain

point $y = \eta$ on the wing, contained in the interval ($y=s$, $y=1$) (fig. 2).

Thus we will obtain the relations

$$q'_{20} = \frac{\mathrm{d}w'_{10}}{\mathrm{d}\eta} \frac{\eta(2 - B^2\eta^2)}{(1 - B^2\eta^2)^{3/2}}, \quad q'_{21} = -\frac{\mathrm{d}w'_{10}}{\mathrm{d}\eta} \frac{1}{(1 - B^2\eta^2)^{3/2}}, \tag{21}$$

which stabilize the dependence from among the intensity of the sources and the variation of the vertical velocities on the thin imaginary wing.

Starting from these relations and keeping in mind (11), we will put the conditions at the limit in the points $\eta = s$ and $\eta = 1$ for the parameter w'_{10} of the vertical velocity:

$$w_{10}^{(1)} - w_{10}^{(0)} = \frac{\overset{*}{q}_{20}}{l} \int_s^l \frac{(l - \eta)(1 - B^2\eta^2)^{3/2}}{\eta(2 - B^2\eta^2)} \mathrm{d}\eta, \tag{22}$$

$$w_{10}^{(1)} - w_{10}^{(0)} = -\frac{\overset{*}{q}_{21}}{l} \int_s^l (l - \eta)(1 - B^2\eta^2)^{3/2} \mathrm{d}\eta, \tag{22'}$$

from where we find the first relations among the constants $\overset{*}{q}_{20}$, $\overset{*}{q}_{21}$, $w_{10}^{(0)}$ and $w_{10}^{(1)}$:

$$\frac{1}{2} \overset{*}{q}_{20} \Big[\sqrt{1 - B^2l^2} - \sqrt{1 - B^2s^2} - \cos h^{-1} \frac{1}{Bl} + \cos h^{-1} \frac{1}{Bs} -$$

$$-\left(1 - \frac{s}{l}\right)\sqrt{1 - B^2s^2} + \frac{1}{Bl}(\sin^{-1} Bl - \sin^{-1} Bs) -$$

$$-\frac{1}{2}\left(\sin^{-1}\frac{1-Bl\sqrt{2}}{\sqrt{2}-Bl}+\sin^{-1}\frac{1+Bl\sqrt{2}}{\sqrt{2}+Bl}-\sin^{-1}\frac{1-Bs\sqrt{2}}{\sqrt{2}-Bs}-\right.$$

$$\left.-\sin^{-1}\frac{1+Bs\sqrt{2}}{\sqrt{2}+Bs}\right)+\frac{1}{\sqrt{2}\,Bl}\left(\sin^{-1}\frac{1-Bl\sqrt{2}}{\sqrt{2}-Bl}-\right.$$

$$\left.\left.-\sin^{-1}\frac{1+Bl\sqrt{2}}{\sqrt{2}+Bl}-\sin^{-1}\frac{1-Bs\sqrt{2}}{\sqrt{2}-Bs}+\sin^{-1}\frac{1+Bs\sqrt{2}}{\sqrt{2}+Bs}\right)\right]=$$

$$=w_{10}^{(1)}-w_{10}^{(0)}, \tag{23}$$

$$\frac{\dot{q}_{21}}{8B}\left[3\left(\sin^{-1}Bl-\sin^{-1}Bs+Bl\sqrt{1-B^2l^2}-Bs\sqrt{1-B^2s^2}\right)+\right.$$

$$+2\left(Bl(1-B^2l^2)^{3/2}-Bs(1-B^2s^2)^{3/2}\right)+\frac{8}{5Bl}\left((1-B^2l^2)^{5/2}-\right.$$

$$\left.\left.-(1-B^2s^2)^{5/2}\right)\right]=w_{10}^{(0)}-w_{10}^{(1)}. \tag{23'}$$

However, for the other parts, the mean vertical vleocity $w_{10}x_1$ or the mean incidence $\alpha_{10}x_1$ of the real wing, equal with that of the first wing components or of the resulting imaginary wings, is obtained taking into consideration the two large wings 2) and 3) compensating reciprocally creating a mean nought slope and we will be able to write the relation

$$\frac{1}{l}\int_0^l w'_{10}(\eta)\,d\eta=w_{10}, \tag{24}$$

only for the thin wing 1.1 Accomplishing the integral, we obtain

$$w_{10}-w_{10}^{(1)}=\frac{\dot{q}_{21}}{48\,B^3l^2}\left\{\frac{8}{5}\left[B(6l-5s)(1-B^2s^2)^{3/2}-\right.\right.$$

$$- Bl(1 - B^2l^2)^{3/2}\Big] - 2\Big[Bl(1 - B^2l^2)^{1/2} - Bs(1 - B^2s^2)^{1/2}\Big] -$$

$$- 3\left(Bl\sqrt{1 - B^2l^2} - Bs\sqrt{1 - B^2s^2} + \sin^{-1}Bl - \sin^{-1}Bs\right)\Big\}. \tag{25}$$

For the determination of the constants A_{21} and Q^*_{21} which appear in expression (13) of the axis velocity, the variation of vertical velocity $w'_{10}x_1$ will be taken into consideration from a point on the wing to the one of nought vertical velocity (for example on the Mach circle). As in previous work [2] - [4], in order to avoid the difficult calculations which appear, we will consider that the liniar distributed sources in the interval (s,l) are concentrated in $y = s'$, for intensities Q_{20} and Q_{21}, in such a manner that we have

$$s' = s + \frac{l - s}{3}, \quad Q_{20} = \frac{1}{2} q^*_{21} l \left(1 - \frac{s}{l}\right)^2, \quad Q_{21} = \frac{1}{2} q^*_{21} l \left(1 - \frac{s}{l}\right)^2, \tag{26}$$

Proceeding in this way, we will write the relations

$$- Im \int_{aripă}^{cercul\ Mach} \frac{\sqrt{1 - B^2x^2}}{x} \frac{d^2\mathcal{U}'_{11}}{dx^2} dx = 0, \tag{27}$$

$$Im \int_{aripă}^{cercul\ Mach} \sqrt{1 - B^2x^2} \frac{d^2\mathcal{U}'_{11}}{dx^2} dx = w^{(1)}_{10}, \tag{27'}$$

where U'_{11} is the axis of disturbance velocity for the simplified case of sources concentrated in $y = s'$, given by the expression

$$\mathcal{U}'_{11} = \frac{1}{x_1}\mathcal{U}'_1 = \frac{A_{21}x}{\sqrt{l^2 - x^2}} + \frac{2}{\pi}(Q_{20} + Q_{21}x)\cos h^{-1}\sqrt{\frac{(l + s')(l - x)}{2l(s' - x)}} -$$

$$- \frac{2}{\pi}(Q_{20} - Q_{21}x)\cosh^{-1}\sqrt{\frac{(l + s')(l + x)}{2l(s' + x)}} + \frac{2}{\pi} Q^*_{21} x \cos h^{-1}\sqrt{\frac{l^2}{x^2}}. \tag{28}$$

Through the accomplishment of the integrals which appear above on the axis ofthe ordinates ($y = 0$, $x = i\mathfrak{z}$) between the limits 0 and ∞ for (27) and on the axis of the abscissas ($\gamma = 0$) between the limits 1 and 1/B for (27'), result

$$A_{21}\frac{(2 - B^2 l^2) E(k) - B^2 l^2 K(k)}{l^2(1 - B^2 l^2)} + \frac{9}{4\pi} q^*_{20}\left(1 - \frac{s'}{l}\right)^2 \frac{\sqrt{l^2 - s'^2}}{s' \, l(1 - B^2 s'^2)}\Big[(1 -$$

$$- B^2 s'^2) E(k) - B^2 l^2 (K(k) - \Pi(q, k))\Big] + \frac{9}{4\pi} l q^*_{21}\left(1 - \frac{s'}{l}\right)^2 \times$$

$$\times \frac{B^2\sqrt{l^2 - s'^2}}{1 - B^2 s'^2}\Big[(2 - B^2 s'^2)\,\Pi(\rho, k) - K(k)\Big] + \frac{2}{\pi} Q^*_{21}\frac{1}{l} \times$$

$$\times \Big[B^2 l^2 K(k) - 2E(k)\Big] = 0, \tag{29}$$

$$Q^*_{21} + \frac{1}{2\sqrt{9 - B^2(l + 2s)^2}}\Big[B^2 l^2\left(1 + 2\frac{s}{l}\right)\left(1 - \frac{s}{l}\right)^2 q^*_{20} + 3l\Big(1 -$$

$$-\frac{s}{l}\Big)^2 q^*_{21}\Big] = - w^{(1)}_{10}, \tag{29'}$$

in which K(k), E(k), Π(p,k) represent the complete eliptical integrals for the first, second and third instances respectively, having the module k and the parameter p given by the relations

$$\Pi(\rho, k) = K(k) + \frac{\sqrt{1 - B^2 s'^2}}{B^2 s'\sqrt{l^2 - s'^2}}\Big[\frac{\pi}{2} - K(k)\,E(\varphi_0, k') + (K(k) -$$

$$- E(k))\,F(\varphi_0, k')\Big], \tag{30}$$

$$k = \sqrt{1 - B^2 l^2},\quad \rho = B^2 s'^2 - 1,\quad k' = Bl,\quad \varphi_0 = \sin^{-1}\frac{s'}{l}. \tag{30'}$$

Due to the separation of flow at the edge of the wing and as a result of the presence of vortexes on the higher side, finite velocities are achieved in those points. Imposing this condition, we will be able to write the relation

$$A_{21} = 0. \tag{31}$$

Eliminating $w_{10}^{(0)}, w_{10}^{(1)}$ and q_{21}^{*} among the equations (23), (23'), (25), (29), (29') and taking into consideration (31), we obtain the constants q_{20}^{*} and q_{21}^{*} :

$$q_{20}^{*} = -\frac{I_{21}}{I_{20}J_{21} - I_{21}J_{20}} w_{10}; \quad q_{21}^{*} = \frac{I_{20}}{I_{20}J_{21} - I_{21}J_{20}} w_{10}, \tag{32}$$

where we made the following notations:

$$I_{20} = 2B\left\{2\left[\sqrt{1 - B^2l^2} - \left(2 - \frac{s}{l}\right)\sqrt{1 - B^2s^2} - \cosh^{-1}\frac{1}{Bl} + \right.\right.$$

$$\left. + \cosh^{-1}\frac{1}{Bs}\right] + \frac{1}{Bl}\left[2(\sin^{-1}Bl - \sin^{-1}Bs) - (Bl - \sqrt{2}) \times \right.$$

$$\times\left(\sin^{-1}\frac{1 - Bl\sqrt{2}}{\sqrt{2} - Bl} - \sin^{-1}\frac{1 - Bs\sqrt{2}}{\sqrt{2} - Bs}\right) -$$

$$\left.\left. - (Bl + \sqrt{2})\left(\sin^{-1}\frac{1 + Bl\sqrt{2}}{\sqrt{2} + Bl} - \sin^{-1}\frac{1 + Bs\sqrt{2}}{\sqrt{2} + Bs}\right)\right]\right\}, \tag{33}$$

$$I_{21} = 3\left(\sin^{-1} Bl - \sin^{-1} Bs + Bl\sqrt{1 - B^2l^2} - Bs\sqrt{1 - B^2s^2}\right) +$$

$$+ 2\,[Bl\,(1 - B^2l^2)^{3/2} - Bs\,(1 - B^2s^2)^{3/2}] + \frac{8}{5Bl}\Big[(1 -$$

$$- B^2l^2)^{5/2} - (1 - B^2s^2)^{5/2}\Big], \tag{33'}$$

$$J_{20} = \frac{9}{8}\,\frac{\frac{l}{s'}\left(1 - \frac{s'}{l}\right)^2}{(B^2l^2K(k) - 2E(k))\sqrt{1 - B^2s'^2}}\left\{\sqrt{\frac{1 - \frac{s'^2}{l^2}}{1 - B^2s'^2}}\Big[(1 -\right.$$

$$\left. - B^2s'^2)\,E(k) - B^2l^2\,(K(k) - \Pi(\rho, k))\Big] - B^2s'^2\,(B^2l^2K(k) - 2E(k))\right\}, \tag{33''}$$

$$J_{21} = \frac{9}{8}\,\frac{l\left(1 - \frac{s'}{l}\right)^2}{(B^2l^2K(k) - 2E(k))\sqrt{1 - B^2s'^2}}\left\{B^2l^2\sqrt{\frac{1 - \frac{s'^2}{l^2}}{1 - B^2s'^2}}\,[(2 -\right.$$

$$\left. - B^2s'^2)\,\Pi(\rho, k) - K(k)] + 2E(k) - B^2l^2K(k)\right\} +$$

$$+ \frac{1}{48\,B^3l^2}\left\{\frac{8}{5}\,[B(6l - 5s)\,(1 - B^2s^2)^{5/2} - Bl(1 - B^2l^2)^{5/2}\,] -\right.$$

$$- 2[Bl(1 - B^2l^2)^{3/2} - Bs(1 - B^2s^2)^{3/2}\,] -$$

$$\left. - 3(Bl\sqrt{1 - B^2l^2} - Bs\sqrt{1 - B^2s^2} + \sin^{-1} Bl - \sin^{-1} Bs)\right\}. \tag{33'''}$$

The constants $\overset{**}{q}_{22}$ and $\overset{**}{q}_{21}$ which appear in the expression U_{1c}, given by (18), are determined taking into consideration the rolè of the third wing component, which will have the mean slope: $-\,w_{10}x_1$. Similarly with (24), we will write

$$\frac{1}{l}\int_0^l w''_{10}(\eta)\,d\eta = -\,w_{10}, \tag{34}$$

from where, taking into consideration (16), we deduce

$$w_{10} - w_{10}^{(1)} = \frac{q_{20}^{**}}{B^2 l^2}\left\{\frac{1}{3}\left[B^2 l^2 \sqrt{1 - B^2 l^2} - 2(1 - \sqrt{1 - B^2 l^2})\right] + \frac{\pi}{4} - \right.$$

$$\left. - \frac{1}{2}\left(\sin^{-1}\frac{1 - Bl\sqrt{2}}{\sqrt{2} - Bl} + \sin^{-1}\frac{1 + Bl\sqrt{2}}{\sqrt{2} + Bl}\right)\right\}, \qquad (35)$$

$$w_{10} - w_{10}^{(1)} = \frac{q_{21}^{**}}{2B^2 l}\left[\frac{1}{3}(1 - B^2 l^2)^{5/2} - \frac{1}{12}(1 - B^2 l^2)^{3/2} - \right.$$

$$\left. - \frac{1}{8}\left(\sqrt{1 - B^2 l^2} + \frac{\sin^{-1} Bl}{Bl}\right)\right]. \qquad (35')$$

Taking into consideration the relation (25), we will obtain from (35) and (35') the constants q_{20}^{**} and q_{21}^{**}:

$$q_{20}^{**} = \frac{q_{21}^{*}}{2B}\left\{\frac{Bl}{15}\left[\left(6 - 5\frac{s}{l}\right)(1 - B^2 s^2)^{5/2} - (1 - B^2 l^2)^{5/2}\right] - \right.$$

$$- \frac{1}{12}\left[Bl(1 - B^2 l^2)^{3/2} - Bs(1 - B^2 s^2)^{3/2}\right] -$$

$$\left. - \frac{1}{8}\left(Bl\sqrt{1 - B^2 l^2} - Bs\sqrt{1 - B^2 s^2} + \sin^{-1} Bl - \sin^{-1} Bs\right)\right\} \times$$

$$\times \left\{\frac{1}{3}\left[B^2 l^2 \sqrt{1 - B^2 l^2} - 2(1 - \sqrt{1 - B^2 l^2})\right] + \right.$$

$$\left. + \frac{1}{2}\left(\frac{\pi}{2} - \sin^{-1}\frac{1 - Bl\sqrt{2}}{\sqrt{2} - Bl} - \sin^{-1}\frac{1 + Bl\sqrt{2}}{\sqrt{2} + Bl}\right)\right\}^{-1}, \qquad (36)$$

$$q_{21}^{**} = \frac{q_{21}^{*}}{Bl}\left\{\frac{Bl}{15}\left[\left(6 - 5\frac{s}{l}\right)(1 - B^2 s^2)^{5/2} - (1 - B^2 l^2)^{5/2}\right] - \right.$$

$$- \frac{1}{12}\left[Bl(1 - B^2 l^2)^{3/2} - Bs(1 - B^2 s^2)^{3/2}\right] - \frac{1}{8}\left(Bl\sqrt{1 - B^2 l^2} - \right.$$

$$\left.\left. - Bs\sqrt{1 - B^2 s^2} + \sin^{-1} Bl - \sin^{-1} Bs\right)\right\}\left[\frac{1}{3}(1 - B^2 l^2)^{5/2} - \right.$$

$$\left. - \frac{1}{12}(1 - B^2 l^2)^{3/2} - \frac{1}{8}\left(\sqrt{1 - B^2 l^2} + \frac{\sin^{-1} Bl}{Bl}\right)\right]^{-1}. \qquad (36')$$

The constant Q^{*}_{21} is deduced immediately from (29), making $A_{21} = 0$:

$$Q^{*}_{21} = -w_{10}^{(0)}. \tag{37}$$

For the determination of the constants Q^{**}_{21} and Q^{***}_{21} we will write, similar with (27'), the following relations:

$$\operatorname{Im} \zeta \int_{\gamma} \sqrt{1 - B^2 x^2}\, \frac{d^2 \mathcal{U}_{1l}}{dx^2}\, dx = w_{10}^{(0)}, \tag{38}$$

$$\operatorname{Im} \zeta \int_{\gamma} \sqrt{1 - B^2 x^2}\, \frac{d^2 \mathcal{U}_{1c}}{dx^2}\, dx = w_{10}^{\prime\prime(0)}, \tag{38'}$$

which will be integrated on a quarter circle γ of very small radius around the origin. Proceeding in this way, we obtain

$$Q^{**}_{21} = -w_{10}^{(0)}, \tag{39}$$

$$Q^{***}_{21} = -w_{10}^{\prime\prime(0)}, \tag{39'}$$

in which $w_{10}^{\prime\prime}(0)$ is the vertical velocity at the middle of the wing compensating for slope which is obtained starting from the relations (16) and (21):

$$w_{10}^{\prime\prime(0)} = -\frac{q^{**}_{21}}{2B^2 l}\left[\frac{1}{15}(1 - B^2 l^2)^{5/2} + \frac{1}{12}(1 - B^2 l^2)^{3/2} + \right.$$
$$\left. + \frac{1}{8}\left(\sqrt{1 - B^2 l^2} + \frac{\sin^{-1} Bl}{Bl}\right) - \frac{2}{5}\right] - w_{10}. \tag{40}$$

4. THE DISTRIBUTION OF PRESSURE AND AERODYNAMIC CHARACTERISTICS

We have shown above that the axis of disturbance velocity of the real wing results through the superpositioning of three imaginary wing components, obtaining formula (19). For the calculation of the distribution of pressure, the total axis velocity given by (19) (fig. 3) will be considered:

$$C_p = -2\frac{u_1}{U_\infty} = -2\,\mathrm{Re}\frac{\mathcal{U}_1}{U_\infty}\,.$$

Moving along to the calculation of the coefficient of lift of the wing, we will make the observation that the wings of symmetrical thickness do not give lift, so that only the coefficient of lift given by "the wing lift" will be taken into consideration:

$$C_z = \frac{8}{3l\,U_\infty}\int_0^l u_{11}\,\mathrm{d}y\,. \tag{41}$$

Taking into consideration (13), we will obtain the following expressions ofthe coefficient of lift of the wing:

$$\begin{aligned} C_z = \frac{8l}{3\pi U_\infty}\Bigg\{&Q^*_{21} + q^*_{20}\left[\left(1-\frac{1}{3}\frac{s}{l}\right)\sqrt{1-\frac{s^2}{l^2}} - \frac{1}{3}\cos^{-1}\frac{s}{l} - \right.\\ &\left.-\frac{s^2}{l^2}\left(1-\frac{2}{3}\frac{s}{l}\right)\cos h^{-1}\frac{l}{s}\right] + q^*_{21}\,l\left[\frac{2}{3}\cos^{-1}\frac{s}{l} - \frac{1}{2}\sqrt{1-\frac{s^2}{l^2}} - \right.\\ &\left.-\frac{s}{l}\left(\frac{1}{3}-\frac{1}{4}\frac{s}{l}\right)\left(\sqrt{1-\frac{s^2}{l^2}} + \frac{s^2}{l^2}\cos h^{-1}\frac{l}{s}\right)\right]\Bigg\}. \end{aligned} \tag{42}$$

The coefficient of the moment is

$$HC_m = \frac{2}{lU_\infty}\int_0^l u_{11}\, y\, \mathrm{d}y = \frac{l^2}{8U_\infty}\left\{\frac{8}{3}Q^*_{21} + q^*_{20}\left[\left(\frac{8}{3}\left(1-\frac{s^2}{l^2}\right) - \frac{s}{l}\left(1-2\frac{s^2}{l^2}\right)\right)\sqrt{1-\frac{s^2}{l^2}} - \cos^{-1}\frac{s}{l}\right] + q^*_{21}\, l\left[2\cos^{-1}\frac{s}{l} - \sqrt{1-\frac{s^2}{l^2}}\left(\frac{8}{5}+\frac{1}{3}\frac{s}{l}\left(1+2\frac{s^2}{l^2}\right)\left(2-\frac{8}{5}\frac{s}{l}\right)\right)\right]\right\}, \tag{43}$$

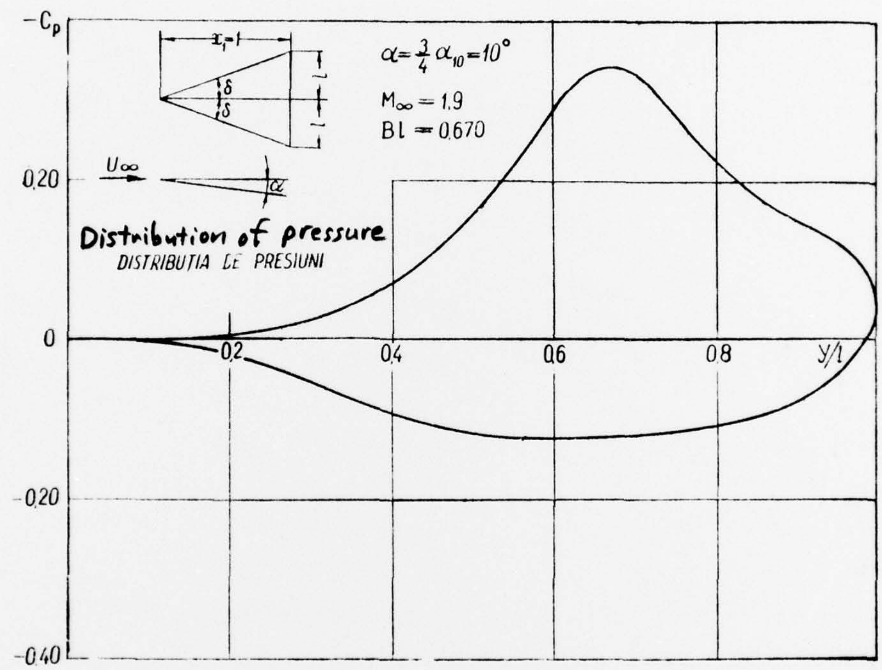

Fig. 3

where H is a length of reference, which can be taken equal with the unit.

As concerns the parameter $\frac{s}{l}$ which enters in the above expressions and which determines the limits of distribution of sources, we will observe that it depends on the position

of the center $\frac{c}{l}$ of the nucleus of the vortex. In previous works (2), (3) we analized in detail the problem of the position of the nucleus of the vortex. In this manner, taking into consideration the experimental results obtained by various authors on the plane delta wing with constant incidence, we will allow for the position of the nucleus of the vortex the following approximate formula of variation with the incidence:

$$\frac{c}{l} \simeq \frac{1}{1 + 1{,}1\,(\alpha)^{1/2}}, \qquad (44)$$

in which α represents the incidence of the wing in the point which would represent the center of gravity of the aerodynamic effects:

$$x_1 = \frac{3}{4}\,. \qquad (45)$$

As has proceeded and in previous works, we will consider the following relation between c and s:

$$c = s + \frac{1}{6}(l - s)\,; \quad \left(\frac{s}{l} = 1{,}2\,\frac{c}{l} - 0{,}2\right). \qquad (46)$$

5. THE PLANE DELTA WING WITH FORCED ANTISYMMETRY

In this case we will analize briefly the flow around thin delta wings with the incidence α equal and of opposed sign on its two halves, taking into consideration the

the separation of flow at the edges.

The formation of vortexes has the direct result of producing a complex field of vertical velocities, which will modify the flow in such a way that the pressures will be finite at the edges.

Proceeding as above, we will chose for the distribution of sources the following liniar functions:

$$q'(\eta)\begin{cases} p_0 \dfrac{\eta}{l} & (-s < \eta < s), \\ \pm q_0\left(1 - \dfrac{|\eta|}{l}\right), & \eta \in [-l, -s] \cup [s, l], \end{cases} \tag{47}$$

in the case of the first two imaginary wing components, and

$$q''(\eta) = k_0 \frac{\eta}{l} \quad (-l < \eta < l), \tag{48}$$

for the wing of symmetrical thickness compensating for slope.

Applying the stability formulas in the exercise of conical motion [1], the axis of disturbance velocities of the three imaginary wings will be the following:

$$\mathcal{U}_l = \frac{A_{11}\, x}{\sqrt{l^2 - x^2}} - \frac{q_0}{\pi}\left[(s - x)\left(1 - \frac{s + x}{2l}\right)\cos h^{-1}\frac{l^2 - s\,x}{l(s - x)} -\right.$$

$$\left. - (s + x)\left(1 - \frac{s - x}{2l}\right)\cos h^{-1}\frac{l^2 + sx}{l(s + x)} + x\sqrt{1 - \frac{x^2}{l^2}}\cos^{-1}\frac{s}{l}\right] +$$

$$+\frac{p_0}{\pi}\left[\frac{s^2-x^2}{2l}\left(\cos h^{-1}\frac{l^2-sx}{l(s-x)}-\cos h^{-1}\frac{l^2+sx}{l(s+x)}\right)+\right.$$

$$\left.+x\sqrt{1-\frac{x^2}{l^2}}\sin^{-1}\frac{s}{l}\right], \qquad (49)$$

for the thin wing, and

$$\mathcal{U}_t=\frac{q_0}{\pi}\left[(l-x)\left(1-\frac{l+x}{2l}\right)\cos h^{-1}\frac{1-B^2lx}{B(l-x)}-\right.$$

$$-(l+x)\left(1-\frac{l-x}{2l}\right)\cos h^{-1}\frac{1+B^2lx}{B(l+x)}-$$

$$-(s-x)\left(1-\frac{s+x}{2l}\right)\cos h^{-1}\frac{1-B^2sx}{B(s-x)}+$$

$$+(s+x)\left(1-\frac{s-x}{2l}\right)\cos h^{-1}\frac{1+B^2sx}{B(s+x)}-$$

$$\left.-\frac{1}{Bl}(\sin^{-1}Bl-\sin^{-1}Bs)\,x\sqrt{1-B^2x^2}\right]+ \qquad (50)$$

$$+\frac{p_0}{\pi}\left[\frac{s^2-x^2}{2l}\left(\cos h^{-1}\frac{1-B^2sx}{B(s-x)}-\cos h^{-1}\frac{1+B^2sx}{B(s+x)}\right)+\right.$$

$$\left.+\frac{x}{Bl}\sqrt{1-B^2x^2}\,\sin^{-1}Bs\right]\pm L,$$

in the case of the wing of symmetrical thickness. For the third wing component, "compensating for slope", we will have the velocity

$$\mathcal{U}_c=\frac{k_0}{2\pi B^2l}\left[B^2(l^2-x^2)\left(\cos h^{-1}\frac{1-B^2lx}{B(l-x)}-\cos h^{-1}\frac{1+B^2lx}{B(l+x)}\right)+\right.$$

$$\left.+2Bx\sqrt{1-B^2x^2}\sin^{-1}Bl\right]. \qquad (51)$$

We will determine the constants q_0, p_0, k_0 in a similar way as has proceeded and in previous work [2] - [4]. Taking into consideration the relation

$$q' = \frac{\eta}{\sqrt{1 - B^2 \eta^2}} \frac{dw'}{d\eta}, \tag{52}$$

known from the theory of conical motion, and by the relations (47), (48), we will put the conditions at the limit for the the vertical velocity of the first wing component:

$$w_0 - w = \frac{p_0}{l} \int_0^s \sqrt{1 - B^2 \eta^2}\, d\eta, \tag{53}$$

$$w_1 - w_0 = q_0 \int_s^l \left(1 - \frac{\eta}{l}\right) \frac{\sqrt{1 - B^2 \eta^2}}{\eta}\, d\eta, \tag{53'}$$

where w_0 and w_1 represent the vertical velocities at the middle, the respective edge of the wing.

Defining a mean vertical velocity equal with that corresponding to the incidence of the real wing, we will write for the thin wing the relation

$$w = \frac{1}{l}\left(w' \eta \Big|_0^s + w' \eta \Big|_s^l - \int_0^s \eta\, dw' - \int_s^l \eta\, dw'\right). \tag{54}$$

The constant A_{11}, which we determine taking into consideration the variation of the vertical velocity from a point on the wing to the one of the nought vertical velocity (the Mach circle),

as in previous work [2], [3], we will cancel it, imposing in this way the condition of nought velocity at the edges. By accomplishing the integral remains

$$\operatorname{Im}\int_{\text{aripă }(0,\,l)}^{\text{cercul Mach }(1/B,\,\infty)} \frac{\sqrt{1-B^2x^2}}{x}\,\mathrm{d}\mathcal{U}_1' = w_1, \tag{55}$$

where U_1' is the axis of disturbance velocity in the simplified case of a concentrated source in $y = s'$ on the thin wing.

Through the accomplishment of the integral (55) on the axis of the abscissa between the limits 1 and 1/B results

$$A_{11} = \frac{2l}{\pi}\left[w_1 + \frac{Q_0}{s'}\left(\sqrt{1-B^2s'^2} - \sqrt{1-\frac{s'^2}{l^2}}\right)\right], \tag{56}$$

in which s' and Q_0 are given by the relations

$$\frac{s'}{l} = \frac{1}{3}\,\frac{q_0\left(1-\frac{s}{l}\right)^2\left(1+2\frac{s}{l}\right) + p_0\frac{s^3}{l^3}}{q_0\left(1-\frac{s}{l}\right)^2 + p_0\frac{s^2}{l^2}}, \tag{57}$$

$$Q_0 = \frac{1}{2l}\left[q_0(l-s)^2 + p_0 s^2\right]. \tag{57'}$$

Similar with (54), we will have in the case of the wing compensating for slope

$$w = -\frac{1}{l}\left(w''\eta\Big|_0^l - \int_0^l \eta\,\mathrm{d}w''\right). \tag{58}$$

Integrating the equations (53), (53'), (54) and (58), we will obtain an algebraic system of equations from which the constants q_0, p_0, k_0 will result:

$$\frac{q_0}{\alpha U_\infty} = \left\{ \frac{3}{2}\left(\sqrt{1 - B^2 s'^2} - \sqrt{1 - \frac{s'^2}{l^2}}\right) \frac{\left[\left(1 - \frac{s}{l}\right)^2 + \frac{p_0}{q_0}\frac{s^2}{l^2}\right]^2}{\left(1 - \frac{s}{l}\right)^2\left(1 + 2\frac{s}{l}\right) + \frac{p_0}{q_0}\frac{s^3}{l^3}} - \right.$$

$$- \frac{1}{2Bl}\left[Bl\sqrt{1 - B^2 l^2} - Bs\sqrt{1 - B^2 s^2} + \sin^{-1} Bl - \sin^{-1} Bs + \right. \tag{59}$$

$$\left. \left. + \frac{2}{3Bl}((1 - B^2 l^2)^{3/2} - (1 - B^2 s^2)^{3/2}) + \frac{2}{3Bl}\frac{p_0}{q_0}(1 - (1 - B^2 s^2)^{3/2})\right] \right\}^{-1}.$$

$$\frac{p_0}{q_0} = 2\left[B(l - s)\sqrt{1 - B^2 s^2} + \sin^{-1} Bl - \sin^{-1} Bs + \frac{1}{3Bl}((1 - B^2 l^2)^{3/2} - \right.$$

$$\left. - (1 - B^2 s^2)^{3/2}) + Bl\left(\cos h^{-1}\frac{1}{Bl} - \cos h^{-1}\frac{1}{Bs}\right)\right]\left[Bs\sqrt{1 - B^2 s^2} + \right. \tag{59'}$$

$$\left. + \sin^{-1} Bs - \frac{2}{3Bl}(1 - (1 - B^2 s^2)^{3/2})\right]^{-1},$$

$$k_0 = -q_0\left\{(1 - B^2 l^2)^{3/2} - (1 - B^2 s^2)^{3/2} + \frac{3}{2}Bl\,[Bl(1 - B^2 l^2)^{1/2} - \right.$$

$$- Bs(1 - B^2 s^2)^{1/2} + \sin^{-1} Bl - \sin^{-1} Bs] + \tag{59''}$$

$$\left. + \frac{p_0}{q_0}[1 - (1 - B^2 s^2)^{3/2}]\right\}[1 - (1 - B^2 l^2)^{3/2}]^{-1}.$$

The coefficient of lift is given only by the first wing component:

$$C_t = \frac{4}{lU_\infty}\int_0^l u_t\,dy = \frac{4l}{\pi U_\infty}\left\{\frac{p_0}{3}\left[\sin^{-1}\frac{s}{l} + \frac{s}{l}\left(2\frac{s^2}{l^2}\cos h^{-1}\frac{l}{s} - \right.\right.\right.$$

$$\left.\left.\left. - \left(1 - \frac{s^2}{l^2}\right)^{1/2}\right)\right] + q_0\left[\left(1 - \frac{1}{3}\frac{s}{l}\right)\left(1 - \frac{s^2}{l^2}\right)^{1/2} - \frac{1}{3}\cos^{-1}\frac{s}{l} - \right.$$

$$\left.\left. - \frac{s_2}{l^2}\left(1 - \frac{2}{3}\frac{s}{l}\right)\cosh^{-1}\frac{l}{s}\right]\right\}, \tag{60}$$

and the coefficient of the moment will be

$$HC_m = \frac{8}{3lU_\infty}\int_0^l u_t\,y\,dy =$$

$$= \frac{l^2}{6U_\infty}\left\{q_0\left[\left(\frac{8}{3}\left(1 - \frac{s^2}{l^2}\right) - \frac{s}{l}\left(1 - 2\frac{s^2}{l^2}\right)\right)\left(1 - \frac{s^2}{l^2}\right)^{1/2} - \cos^{-1}\frac{s}{l}\right] - \right.$$

$$\left. - p_0\left[\frac{s}{l}\left(1 - 2\frac{s^2}{l^2}\right)\left(1 - \frac{s^2}{l^2}\right)^{1/2} - \sin^{-1}\frac{s}{l}\right]\right\}. \tag{61}$$

Concerning the position of the nucleus of the vortex in the case of plane delta wings with forced antisymmetry, the same formula is taken as in previous works:

$$\frac{c}{l} \simeq \frac{1}{1 + 1{,}7\,(\alpha)^{1/2}}, \tag{62}$$

$$\frac{s}{l} = 1{,}2\,\frac{c}{l} - 0{,}2\,. \tag{62'}$$

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